Calculus Warm Up # 6-1

Test for convergence. (Use partial fractions and telescoping series test)

$$\sum_{n=1}^{\infty} \frac{4}{n(n+2)}$$

10.8 Power Series

Determining Domain

Radius of Convergence

Interval of Convergence

Power Series centered at x = 0 or x = c

(c = constant)

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_n x^n + \ldots$$

$$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + \ldots + a_n(x-c)^n + \ldots$$

*To simplify notation, it is agreed that $(x-c)^0 = 1$,

even if x = c.

$$0^0 = 1$$

Power Series

Ex: centered at x = 0

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

Ex: centered at x = 1

$$\sum_{n=0}^{\infty} \frac{1}{n}(x-1)^n = (x-1) + \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 + \ldots$$

*To simplify notation, it is agreed that $(x-c)^0 = 1$, even if x = c.

Power Series written as a function of x:

$$f(x) = \sum_{n=0}^{\infty} a_n(x-c)^n$$

Domain: the set of all x for which the series converges.

Determining the Domain of a Power Series

1) A single point (the center)
2) An interval centered at c
3) All reals

Domain of a Power Series

1) A single point (the center)
2) An interval centered at c
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Radius of Convergence, R.

1) Series converges only at c: $R = 0$
2) If there is a real number, $R > 0$ such that for $|x - c| < R$, the series converges absolutely, $|x - c| > R$, it diverges.
3) Series converges for all reals, $R = \infty$

Using the ratio test to find the radius of convergence:

Ex: $\sum_{n=0}^{\infty} n!x^n$ centered at x = 0

Ratio Test:

$$\lim_{n\to\infty}\left|\frac{(n+1)!\,x^{n+1}}{n!\,x^n}\right|$$

$$\lim_{n\to\infty}|(n+1)|\,|x| = \infty \text{ which is } > 1$$

$\therefore$ Series diverges for $|x| > 0$

It converges only at its center, 0

The radius of convergence: $\boxed{R=0}$

Find the radius of convergence:

Ex: $\sum_{n=0}^{\infty} 3(x-2)^n$ centered at x = 2,

Ratio Test for $x \neq 2$

$$\lim_{n\to\infty}\left|\frac{3(x-2)^{n+1}}{3(x-2)^n}\right| = \lim_{n\to\infty}|x-2| = |x-2|$$

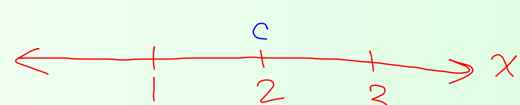
$\therefore$ Series converges for $|x-2| < 1$

and diverges for $|x-2| > 1$

$|x-2| < 1$

$-1 < x-2 < 1$

$1 < x < 3$



$\boxed{R=1}$

Finding an interval of convergence:

Ex: $\sum_{n=1}^{\infty} \frac{x^n}{n}$ centered at x = 0,

Ratio Test: $\lim_{n\to\infty} \left| \frac{x^{(n+1)}}{(n+1)} \cdot \frac{n}{x^n} \right|$

$= \lim_{n\to\infty} \left| \frac{n}{n+1} \right| |x| = |x|$

$\boxed{R=1}$

interval of convergence: $(-1, 1)$, but we need to consider the endpts...

@ $x=1$: $\sum_{n=1}^{\infty} \frac{1}{n}$ divergent harmonic p-series

@ $x=-1$: $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} = -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} \cdots$ converges.

$\therefore$ Interval of convergence $= [-1, 1)$

You Try: Find the interval of convergence for:

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x+1)^n}{2^n}$$

$\lim_{n\to\infty} \left| \frac{(x+1)^{n+1}}{2^{n+1}} \cdot \frac{2^n}{(x+1)^n} \right|$

$= \lim_{n\to\infty} \left| \frac{x+1}{2} \right| = \left| \frac{x+1}{2} \right|$

If $\left| \frac{x+1}{2} \right| < 1$, converges

$-1 < \frac{x+1}{2} < 1$

$\boxed{-3 < x < 1}$ ← interval of convergence

$\boxed{R=2}$

@ $x = 1$ & -3 both diverge

One more:

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x)^{2n+1}}{(2n+1)!}$$

$$\lim_{n\to\infty} \left| \frac{x^{2(n+1)+1}}{(2(n+1)+1)!} \cdot \frac{(2n+1)!}{x^{2n+1}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(2n+1)!}{(2n+3)!} \cdot \frac{x^{2n+3}}{x^{2n+1}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{x^2}{(2n+3)(2n+2)} \right|$$

$= 0$ for all x

so $R = \infty$

HW: p. 623, # 1 - 15 odd

(Finish the Yellow FR Practice)

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

a) $g(1) = \int_1^1 f(t)\,dt = 0$

$g'(x) = \frac{d}{dx}\int_1^x f(t)\,dt$

$g'(x) = f(x)$

$g'(1) = 0$

b) $g'(x) = f(x)$

g increasing where $f(x) > 0$ on $(1, \pi)$

c) when $g' = 0$ & goes from $-$ to $+$ or @ endpt $x = 2\pi$

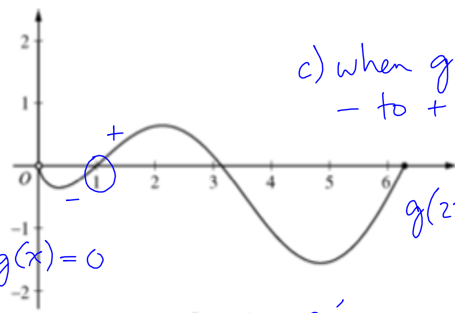
$g(1) = 0$

$g(2\pi) = \underbrace{\int_1^{\pi} f(t)\,dt + \int_{\pi}^{2\pi} f(t)\,dt}$

More negative area so $g(2\pi) < 0$

Ab. Min @ $x = 2\pi$

d) g tangent to x-axis where $g(x) = 0$ and $g'(x) = 0$ at $x = 1$



Graph of f $= g'$

4. Let f be the function given by $f(x) = (\ln x)(\sin x)$. The figure above shows the graph of f for $0 < x \le 2\pi$. The function g is defined by $g(x) = \int_1^x f(t)\,dt$ for $0 < x \le 2\pi$.

(a) Find $g(1)$ and $g'(1)$.

(b) On what intervals, if any, is g increasing? Justify your answer.

(c) For $0 < x \le 2\pi$, find the value of x at which g has an absolute minimum. Justify your answer.

(d) For $0 < x < 2\pi$, is there a value of x at which the graph of g is tangent to the x-axis? Explain why or why not.

5. Let f be the function satisfying $f'(x) = 4x - 2xf(x)$ for all real numbers x, with $f(0) = 5$ and $\lim_{x\to\infty} f(x) = 2$.

$f'(-0.5) = 4(-0.5) - 2(-0.5)[f(-0.5)]$

$= -2 + 1(5)$

$= 3$

(a) Find the value of $\int_0^{\infty} (4x - 2xf(x))\,dx$. Show the work that leads to your answer.

(b) Use Euler's method to approximate $f(-1)$, starting at $x = 0$, with two steps of equal size.

(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 4x - 2xy$ with the initial condition $f(0) = 5$.

a) $\lim_{b\to\infty} \int_0^b f'(x)\,dx$

$= \lim_{b\to\infty} [f(x)]_0^b$

$= \lim_{b\to\infty} [f(b) - f(0)]$

$= 2 - 5 = \boxed{-3}$

b) $\Delta x = -0.5$ start @ $(0, 5)$

$f(-0.5) \approx f(0) + \underbrace{(-0.5)f'(0)}_{\Delta y}$

$\approx 5 + 0$

$f(-1) \approx 5 + (-0.5)[f'(-0.5)]$

$\approx 5 + \left(-\frac{3}{2}\right)$

$\boxed{f(-1) \approx \frac{7}{2}}$

5c) $\frac{dy}{dx} = 2x(2-y)$

$\int \frac{1}{2-y}\,dy = \int 2x\,dx$

$-\ln|2-y| = x^2 + c$

$\ln|2-y| = -x^2 + C$

change to exponent form:

$e^{(-x^2+c)} = 2-y$

$e^{-x^2} \cdot e^{c} = 2-y$

(e^c is just a constant)

$Ce^{-x^2} = 2-y$

$Ce^{0} = 2-5$ (plug in (0,5))

$C = -3$ plug into

$-3e^{-x^2} = 2-y$

$\boxed{y = 2 + 3e^{-x^2}}$

6. The function g is continuous for all real numbers x and is defined by

$$g(x) = \frac{\cos(2x) - 1}{x^2} \text{ for } x \neq 0.$$

(a) Use L'Hospital's Rule to find the value of $g(0)$. Show the work that leads to your answer.

(b) Let f be the function given by $f(x) = \cos(2x)$. Write the first four nonzero terms and the general term of the Taylor series for f about $x = 0$.

(c) Use your answer from part (b) to write the first three nonzero terms and the general term of the Taylor series for g about $x = 0$.

(d) Determine whether g has a relative minimum, a relative maximum, or neither at $x = 0$. Justify your answer.

a) $g(0) = \lim_{x \to 0}\left[\frac{\cos 2x - 1}{x^2}\right] = \frac{0}{0}$

$= \lim_{x \to 0}\left[\frac{-\cancel{2}\sin 2x}{\cancel{2}x}\right] = \frac{0}{0}$

$= \lim_{x \to 0}\left[\frac{-2\cos 2x}{1}\right] = \boxed{-2}$

b) ...

more later.

I'm hungry and need to go home!